

Heat flux and acoustic power in a convection-driven T-shaped thermoacoustic system

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ABSTRACT

The present work considers a convection-driven T-shaped standing-wave thermoacoustic system. To gain insights on the conversion process of heat to sound and to study the nonlinear coupling between unsteady heat release and acoustic disturbances, thermodynamic analysis, numerical and experimental investigations are conducted. Three parameters are examined: (1) the inlet flow velocity, (2) heater temperature and (3) heat source location. Their effects on triggering limit cycle oscillations are first investigated in 2D numerical model. As each of the parameters is varied, the heat-driven acoustic signature is found to change. The main nonlinearity is identified in the heat fluxes. To characterize the transient (growing) behavior of the pressure fluctuation, the thermoacoustic mode growth rate is defined and calculated. It is found that the growth rate decreases first and then 'saturates'. Similar behavior is observed by examining the slope of Rayleigh index. Furthermore, the overall efficiency of converting the input thermal energy into acoustical energy is defined and calculated. It is found that the energy conversion efficiency can be increased by increasing the inlet flow velocity. To validate our numerical findings, a cylindrical T-shaped duct made of quartz-glass with a metal gauze attaching on top of a Bunsen burner is designed and tested. Supercritical bifurcation is observed. And the experimental measurements show a good agreement with the numerical results in terms of mode frequency, mode shape, sound pressure level and Hopf bifurcation behavior.

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1. Introduction

The heat-to-sound or visa versa conversion mechanism (also known as thermoacoustics) has fascinated scientists for many years now, due to its practical energy applications [1]. Standing-wave engines is one of the simplest thermoacoustic system, which operate with a standing acoustic wave in a resonance tube [2,3]. The acoustic waves causes pressure oscillations as well as gas displacement [4]. One of the typical types of resonance tubes is Rijke-type combustor [5,6]. It is a straight vertical tube with a heat source enclosed in its lower half. It provides an elementary example of convection-driven thermoacoustic oscillations [2], which result from a feedback loop between the heat transferred to the fluid and the acoustics propagating in the tube. If the heat input is at times of high pressure, a self-amplification of acoustic fluctuations may lead to a limit cycle [7,8]. The limit cycle is mainly due to the nonlinear effects, which cannot be predicted by linear theory [5].

To elucidate or optimize the conversion process in Rijke-type thermoacoustic systems, extensive investigations [8–13] have

been conducted. The pressure power spectrum of a Rijke combustor [12] is experimentally measured to study the nonlinear coupling between heat release rate and the heat-driven sound. The coupling process is shown to play an important role in triggering thermoacoustic oscillations. The transition to Rijke-type thermoacoustic oscillations is experimentally investigated [8]. To predict the stability behaviors of a Rijke tube, theoretical modal stability analysis [9] is conducted using Green's function approach. It shows that the thermoacoustic frequency is oscillating with increased 'heater' location, which is experimentally validated [13]. In addition, numerical simulations of Rijke-type thermoacoustic oscillations are performed by using FLOW3D [11] or FLUENT [10]. The oscillations result from the proper 'phasing' between the heat transferred to the surrounding fluid and the acoustics of the resonator.

Different designs and various forms of input energy in thermoacoustic systems have been tested [14–22] to elucidate the heat-driven large-amplitude or high-frequency sound waves [14,15,23,24]. By using an adjustable-power electrical heater, a miniature thermoacoustic Stirling engine is designed and tested [16]. Thermoacoustic converters powered by engine waste heat [17] or solar power [18] have also been developed. Recent developments and application of thermoacoustic system for energy harvesting are experimentally investigated [3,6]. It is shown that the

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Nomenclature

A	bottom branch cross-sectional area, m ²	$\mathcal{T}$	oscillation period, s
A_l, A_r	cross-sectional areas of left and right branches, m ²	$\mathbf{u}(\mathbf{x}, t)$	velocity at a given point $\mathbf{x}$, m/s
A_f	flame surface area, m ²	u_i	inlet velocity in i direction, m/s
A_s	heater surface area, m ²	$\mathfrak{B}$	thermal and viscous damping, J
A_0	initial amplitude, m/s	$\mathbf{x}$	coordinates of a given point
$\mathfrak{B}$	mechanical work done by the fluid supporting a normal stress, J	x	axial position, m
$\bar{c}$	speed of sound in air, m/s	x_f	heat source location along x_2 , m
c_p, c_v	heat capacity ratio at constant pressure and volume, kJ/kg K	y	horizontal axial position, m
D	the diameter of the bottom stem, m	Greek symbols	
$\mathcal{E}_a$	generated sound power, J	α	pressure fluctuation amplitude slope, Pa/s
$\mathfrak{E}_a$	acoustical energy, J	δ	Kronecker delta function
$\mathfrak{E}_e$	exchange rate from heat to sound, J	Δq_r	the heat of reaction per unit mass of the mixture, J/kg
$\mathcal{G}$	exponential complex function	ΔT_s	the surface temperature difference, K
h	convection coefficient, W/m ² K	ΔT_f	the temperature difference between the fluid and the heater surface, K
L_l, L_r	left and right branch length, m	γ	the ratio of specific heat
$\mathfrak{L}_f$	the thermal length scale in the fluid, m	λ	conduction coefficient, W/m K
$\mathfrak{L}_s$	the thermal length scale in the heating band, m	μ	viscosity, kg/m s
M_w	molecular weight, kg/mol	ω	oscillation frequency, rad/s
$\mathbf{n}$	the unit vector of the direction normal to the enclosed surface	ρ	air density, kg/m ³
p	instantaneous pressure, Pa	ρ_m	the fuel–air mixture density, kg/m ³
p_i, p_o	inlet and outlet pressure, Pa	τ	the viscous stress tensor, N/m
q	heat flux, W/m ²	Ψ	energy conversion efficiency
Q	heat production per unit volume J/m ³	σ	thermoacoustic oscillation growth rate, s ^{−1}
$\dot{Q}$	unsteady heat release rate, kJ/s	Subscript	
Q_t	input heat power, Watt	s	heating band surface
R	gas constant, J/mol K	a ,	acoustic
$\mathcal{R}$	radius of the tube, m	o	branch outlet
$\mathfrak{R}_u$	universal gas constant, kJ/kmol K	Superscript	
$\mathfrak{R}_a$	normalized Rayleigh index	$-$	mean value
S	entropy per unit mole, J/mol K	$\cdot$	time derivative
s	specific entropy, J/kg K	$'$	fluctuation part
S_u	flame speed, m/s		
Ξ	viscous stress, Pa		
T	Temperature, K		

convection-driven Rijke-Zhao system can harvest 60% more power than the conventional conduction-driven standing-wave system [3]. The standing-wave engine energy conversion efficiency could be increased dramatically by using high-pressure gas, such as 13.8 bar helium [25]. However, such engines work less efficient than traveling-wave (Stirling) engines. It is experimentally demonstrated [26] that a traveling-wave engine can achieve 49% of the Carnot efficiency. It is also shown [27] that as high as 18% thermal-to-electric conversion efficiency is achievable by replacing piezoelectric generator with electrodynamic alternator.

There are current interests in more complex geometric configuration of convection-driven thermoacoustic systems. However, few designs and investigations are reported. In addition, to optimize the conversion from heat to sound, the dynamic interaction between unsteady heat release [1,2] and acoustic waves needs to be better understood. Thus there is a need to identify the roles of the heat source and flow conditions, and to investigate the effect of its position and the heat fluxes in triggering thermoacoustic oscillations, especially in a complex geometric system. Lack of these investigations in the literature partially motivated the present work.

In this work, a convection-driven T-shaped thermoacoustic system is considered. Both experimental and numerical simulations are conducted. To gain insight on the heat-to-sound conversion

mechanism and the nonlinearity, a general heat-driven acoustic wave equation is derived by performing thermodynamic analysis. This is described in Section 2. The unsteady Navier–Stokes equations are iteratively solved by applying a finite volume method on spatially discretized mesh of the T-shaped system, as described in Section 3. The effects of the inlet flow velocity, heat source location and heater temperature on triggering thermoacoustic oscillations are then examined. This is described in Section 4. The thermoacoustic oscillation growth rate, Rayleigh index, and the heat-to-sound energy conversion efficiency are defined and estimated. Finally, experiments are conducted to validate our numerical findings. This is described in Section 5. Comparison is then made between the numerical and experimental results.

2. Thermodynamic analysis

2.1. Heat-driven acoustic model

To shed light on thermoacoustic energy conversion, a general heat-driven acoustic wave equation is derived by conducting a generalized thermodynamic analysis [28]. A T-shaped thermoacoustic system is considered, as shown in Fig. 1(a). It consists of using multiple concentric constant-temperature rings as a heat

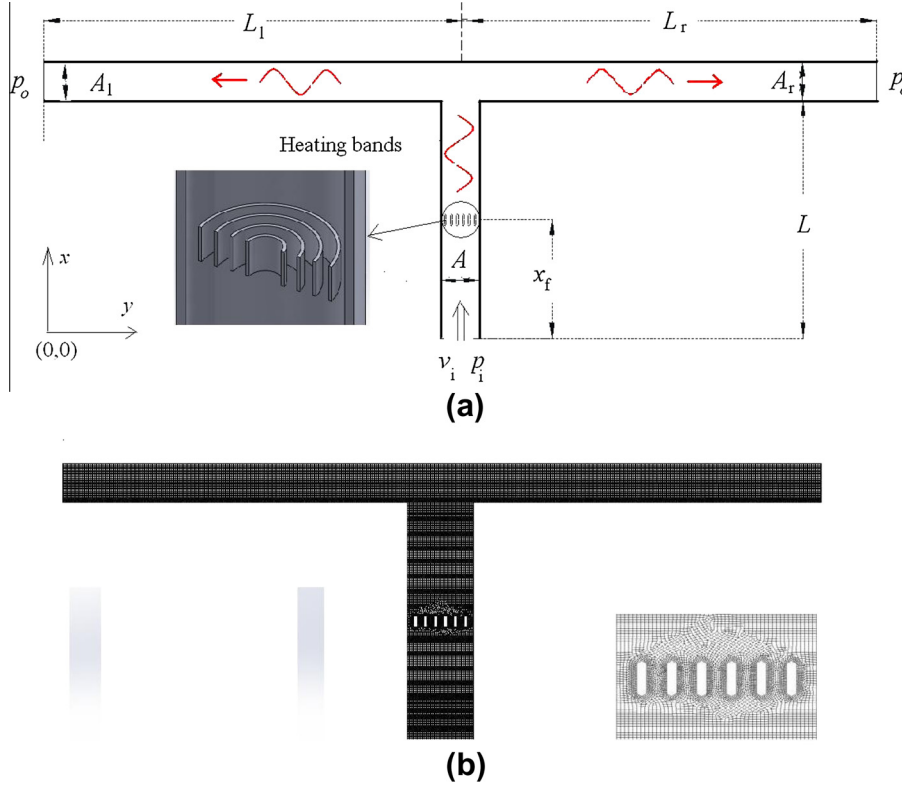


Fig. 1. Schematic of a convection-driven T-shaped standing-wave thermoacoustic system.

source placed in the bottom stem of the tube. As the air is heated up, a buoyancy-driven convection flow with a low Mach number $\bar{M}^2 \ll 1$ is generated. Thus the fluid considered is assumed to satisfy the perfect gas laws. The state equation of perfect gas [29] is given as

$$\frac{p}{\rho} = RT = \frac{\mathfrak{R}_u}{M_w} T = \frac{c^2}{\gamma} \quad (1)$$

where p is the pressure, ρ density, R is gas constant and T is the temperature, $\mathfrak{R}_u = 8.3143$ kJ/kmol K is the universal constant, $\gamma = 1.4$ is the ratio of specific heat, c is the sound speed, and M_w is the molecular weight. The pressure, velocity, density and temperature of the flow at a given location $\mathbf{x} = (x_1, x_2, x_3) = (x, y, z)$ and time consist of a mean part denoted by an overbar, and a fluctuating one denoted by a prime as given as

$$p(\mathbf{x}, t) = \bar{p}(\mathbf{x}) + p'(\mathbf{x}, t) \quad \mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t) \quad (2)$$

$$\rho(\mathbf{x}, t) = \bar{\rho}(\mathbf{x}) + \rho'(\mathbf{x}, t) \quad T(\mathbf{x}, t) = \bar{T}(\mathbf{x}) + T'(\mathbf{x}, t) \quad (3)$$

where velocity $\mathbf{u}(\mathbf{x}, t) = (u_1, u_2, u_3)$.

If the internal energy of the fluid mixture by e , then the first law of thermodynamics considering energy conservation can be described by

$$\frac{de}{dt} + \frac{pd(\rho^{-1})}{dt} = T \frac{ds}{dt} \quad (4)$$

Now consider a control volume enclosing the heating rings, the density ρ varies through changes in both the pressure p and specific entropy s . Thus

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial p} \frac{dp}{dt} + \frac{\partial \rho}{\partial s} \frac{ds}{dt} = \frac{1}{c^2} \frac{dp}{dt} + \frac{\partial \rho}{\partial s} \frac{ds}{dt} \quad (5)$$

For a perfect gas $\rho = p/RT = \gamma p/c^2$, $\partial \rho / \partial s|_p = -\rho T(\gamma - 1)/c^2 = -\rho/c_p$ and so Eq. (5) can be rewritten as

$$\frac{d\rho}{dt} = \frac{1}{c^2} \frac{dp}{dt} - \frac{\rho(\gamma - 1)}{c^2} T \frac{ds}{dt} = \frac{1}{c^2} \frac{dp}{dt} - \frac{(\gamma - 1)}{c^2} Q \quad (6)$$

where $Q = \rho T ds/dt$ is the heat addition per unit volume. And it consist of a mean and fluctuating part as $Q = \bar{Q} + Q'$.

The mass and momentum equations over the control volume, when linearized with respect to the perturbation quantities, can be written as

$$\frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial u'_i}{\partial x_i} = 0 \quad (7)$$

$$\bar{\rho} \frac{\partial u'_i}{\partial t} + \frac{\partial}{\partial x_i} (p' \delta_{ij} - \Xi) = 0 \quad (8)$$

where Ξ denotes the 'friction'/damping (e.g. viscous stress) effect. By substituting linearized Eq. (6) into Eq. (7), multiplying Eq. (7) with $\bar{c}^2 \rho'$, Eq. (8) with u'_i and addition, an energy conservation equation is obtained namely,

$$\frac{\partial}{\partial t} \left[\frac{1}{2} \bar{\rho} u_i'^2 + \frac{1}{2} \frac{\bar{c}^2 \rho'^2}{\bar{\rho}} \right] + \frac{\partial}{\partial x_i} (p' \delta_{ij} u'_i) - u'_i \frac{\partial \Xi}{\partial x_i} = \frac{(\gamma - 1)}{\gamma \bar{p}} p' Q' \quad (9)$$

Integrating Eq. (9) over the entire system leads to

$$\frac{\partial}{\partial t} \iiint \left[\frac{1}{2} \frac{\bar{c}^2 \rho'^2}{\bar{\rho}} + \frac{1}{2} \bar{\rho} u_i'^2 \right] d\mathbf{x}_i = \iiint \frac{(\gamma - 1)}{\gamma \bar{p}} p' Q' d\mathbf{x}_i - \mathfrak{B} - \mathfrak{W} \quad (10)$$

where

$$\mathfrak{B} = \iiint \frac{\partial}{\partial x_i} (p' \delta_{ij} u'_i) d\mathbf{x}_i \quad \mathfrak{W} = \iiint -u'_i \frac{\partial \Xi}{\partial x_i} d\mathbf{x}_i \quad (11)$$

It can be seen from Eq. (10) that the term on the left hand side describes the classical acoustic disturbance energy $\mathfrak{E}_a$ as defined as

$$\mathfrak{E}_a = \iiint \left[\frac{1}{2} \frac{\bar{c}^2 \rho'^2}{\gamma \bar{p}} + \frac{1}{2} \bar{\rho} u_i'^2 \right] d\mathbf{x}_i \quad (12)$$

The first term on right hand side of Eq. (12) describe the potential energy $\mathcal{E}_a^p$. The other term describes the kinetic energy i.e. $\mathcal{E}_a^k = \bar{\rho} u_i^2 / 2$.

The first term on the right-hand-side of Eq. (10) as defined as $\mathcal{E}_e$

$$\mathcal{E}_e = \iiint \frac{(\gamma - 1)}{\gamma \bar{p}} p' Q' dx_i \quad (13)$$

describes the energy exchange between sound and heat input. The second term $\mathcal{B}$ describes the mechanical work done by the fluid supporting an externally induced normal stress p' and moving with velocity u_i' . The last term $\mathcal{W}$ is associated with thermal and viscous dissipation. Substituting Eqs. (12) and (13) into Eq. (10) and integrating it over one period leads to

$$\int_t^{t+T} \mathcal{E}_e dt = \mathcal{E}_a|_t^{t+T} + \int_t^{t+T} [\mathcal{B} + \mathcal{W}] dt \quad (14)$$

Eq. (14) reveals that the input thermal energy is partially converted into acoustical energy, partially dissipated by thermal and viscous effect, and partially used to produce mechanical work. When there is no heat addition, $Q = 0$ (isentropic), and the working fluid is quiescent (stagnant and uniform) and frictionless, Eq. (10) can be simplified into the classical acoustic energy balance equation:

$$\frac{\partial}{\partial t} \mathcal{E}_a = -\mathcal{B} = - \iint_A \mathbf{I} \cdot \mathbf{n} dA \quad (15)$$

where $\mathbf{I} = p' \mathbf{u}'$ is the acoustic intensity. A is the surface enclosing the control volume with outer normal $\mathbf{n}$. Here the Gauss theorem is used to transform $\iiint_V (\nabla \cdot \mathbf{I}) dV$ into the surface integral $\iint_A (\mathbf{I} \cdot \mathbf{n}) dA$. And it can be used to calculate the acoustic power generated by heat addition.

To produce thermoacoustic oscillations (heat-driven sound), one of the necessary condition as suggested by Eq. (14) is

$$\mathcal{E}_e > 0, \quad \text{equivalently} \quad -\frac{\pi}{2} < \angle p' Q' < \frac{\pi}{2} \quad (16)$$

that is to say p' and Q' are in phase. If $|\angle p' Q'| > 90^\circ$, i.e. p' and Q' are out of phase, then the energy exchange rate $\mathcal{E}_e < 0$. And no sound

will be produced, even with large amount of heat being added. This is known as 'Rayleigh criterion'. The other critical condition is that

$$\int_t^{t+T} \mathcal{E}_e dt > \int_t^{t+T} [\mathcal{B} + \mathcal{W}] dt \quad (17)$$

That is to say the heat exchange over one period must be greater than the sum of that dissipated due to thermal and viscous effects and that used to generate mechanical work. To optimize heat-to-sound conversion, it is necessary to maximize the difference between the terms on the left- and right-hand-side in Eq. (17). This can be achieved by maximizing $\mathcal{E}_e$ and minimizing $\mathcal{W}$.

By neglecting the damping term $\mathcal{W}$, substituting Eq. (6) into Eq. (7) differentiating Eq. (7) with respect to time t , taking divergence of Eq. (8), an inhomogeneous heat-driven wave equation is obtained namely,

$$\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \bar{\rho} \nabla \cdot \left(\frac{1}{\bar{\rho}} \nabla p' \right) = \frac{\gamma - 1}{c^2} \dot{Q}'(t) \quad (18)$$

where overdot denotes the time derivative. Eq. (18) describes a general heat-driven acoustic wave equation. $\dot{Q}'(t)$ describes the time rate of heat addition from a heat source. Typically, a flame or an electrical heater is used. The instantaneous heat release rate Q from such heat sources [5,29,30] can be estimated as

$$Q(t) = \begin{cases} \rho_m S_u A_f \Delta q_r & \text{flame} \\ \lambda \partial T / \partial x A_s & \text{heater} \end{cases} \quad (19)$$

where ρ_m is the fuel-air mixture density, S_u is the flame speed, A_f is the flame surface area, and Δq_r is the heat of reaction per unit mass of the mixture, λ is the thermal conductivity, $\partial T / \partial x$ is the spatial temperature gradient between the heater and surrounding air, A_s is the surface area of the heater.

3. T-shaped thermoacoustic model

The T-shaped numerical configuration is generated with structural mesh in Pointwise [31], as shown in Fig. 1(b). Multiple con-

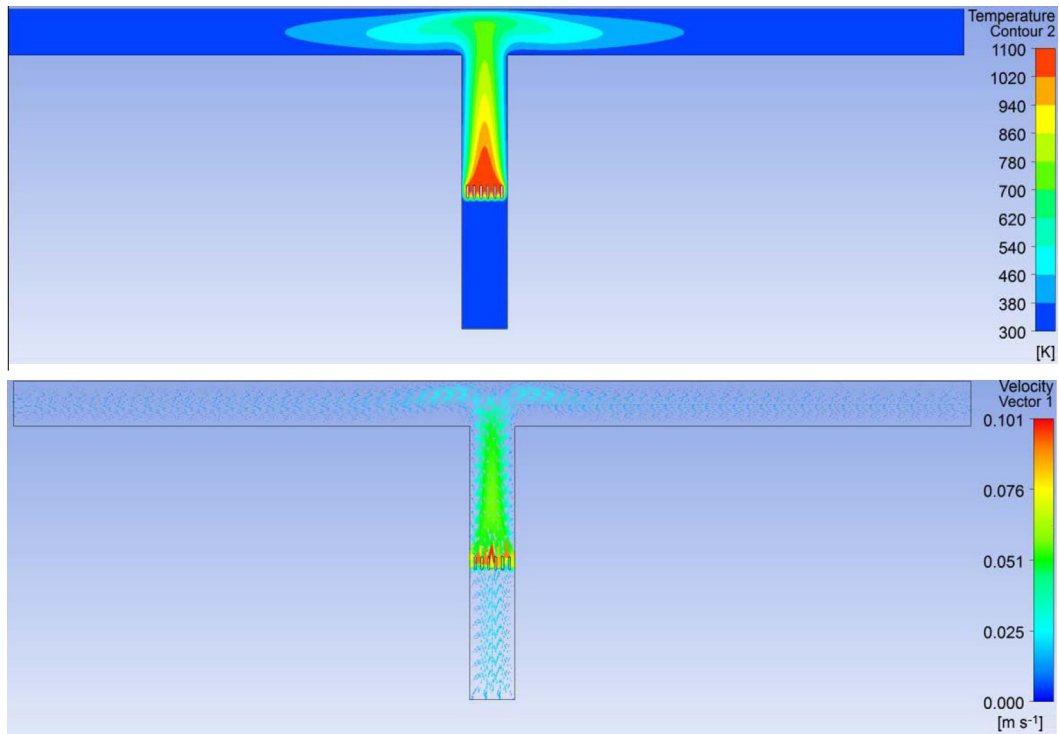


Fig. 2. Temperature and velocity contours of the T-shaped thermoacoustic system.

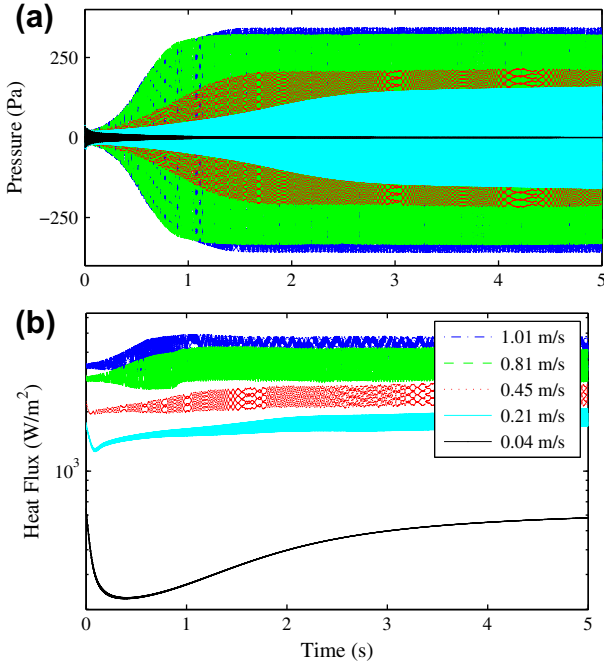


Fig. 3. Variation of pressure and heat flux fluctuations with time, as $x_f/L = 0.5$, $T_s = 1100$ K and v_i is set to 5 different values.

centric rings with constant surface temperature are equally spaced along y-axis. They have a length of $L_h = 5$ cm and placed at x_f . Near the heat sources, finer grid (Hexahedra) is used. The geometric dimensions are $L_l = L_r = 500$ mm, $L = 300$ mm, $A_l = A_r = \pi R^2 = 1.6 \times 10^{-3}$ m², $A = 2.0 \times 10^{-3}$ m². Before starting the unsteady simulations to capture the thermoacoustic oscillations generated in the T-shaped system, a steady solution is first obtained in terms of the temperature and velocity contours, as shown in Fig. 2. The boundary conditions are set according to our experimental setup as shown later as: the bottom stem (mother tube) open end is pressure inlet with a gauge pressure 0.01 Pa, while the bifurcating

branches open ends are pressure outlets $p'_o = 0$ Pa. The heating band surface temperature is set to 1100 K, which is closed to the heat source temperature used in our experiment described later. The heat addition causes a convection flow moving towards the outlets.

The steady conservation equations of mass, momentum and energy with equation of state for ideal gas are solved iteratively for a laminar 2D flow as given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (20a)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_i} (-p \delta_{ij} + \tau_{ij}) \quad (20b)$$

$$\frac{\rho c_p}{\gamma} \left(\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) + p \frac{\partial u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\lambda \frac{\partial T}{\partial x_j} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j} + Q \quad (20c)$$

where u_i the velocity component of $\mathbf{u}(\mathbf{x}, t)$ in i th direction, and τ_{ij} is the viscous stress tensor as given as

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2\mu}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (21)$$

δ_{ij} is the Kronecker delta function. When $i = j$, $\delta_{ij} = 1$. Otherwise $\delta_{ij} = 0$. μ is the dynamic viscosity, and λ is the coefficient of heat addition to the fluid particle. Q is the instantaneous heat production per unit volume from the constant-temperature heating bands. The heat transfer rate via conduction over the heating band can be shown as

$$\dot{Q} = \lambda \mathcal{A}_s \frac{\partial T_s}{\partial x_i} = \sum_{i=1}^N \lambda \mathcal{A}_{s,i} \frac{\partial T_s}{\partial x_i} \quad (22)$$

where $\mathcal{A}_s$ is the total surface area of the heating bands. $\mathcal{A}_{s,i}$ denotes the surface area of i th heating cell. As the surrounding fluid flows, convection heat transfer is expected. And it is balanced by the conduction heat transfer as given as

$$\dot{Q} = h \mathcal{A}_s (T_f - T_s) = h \mathcal{A}_s \Delta T_f = q \mathcal{A}_s \quad (23)$$

where q is the heat flux. It can be shown that the convection heat transfer coefficient h can be determined by using

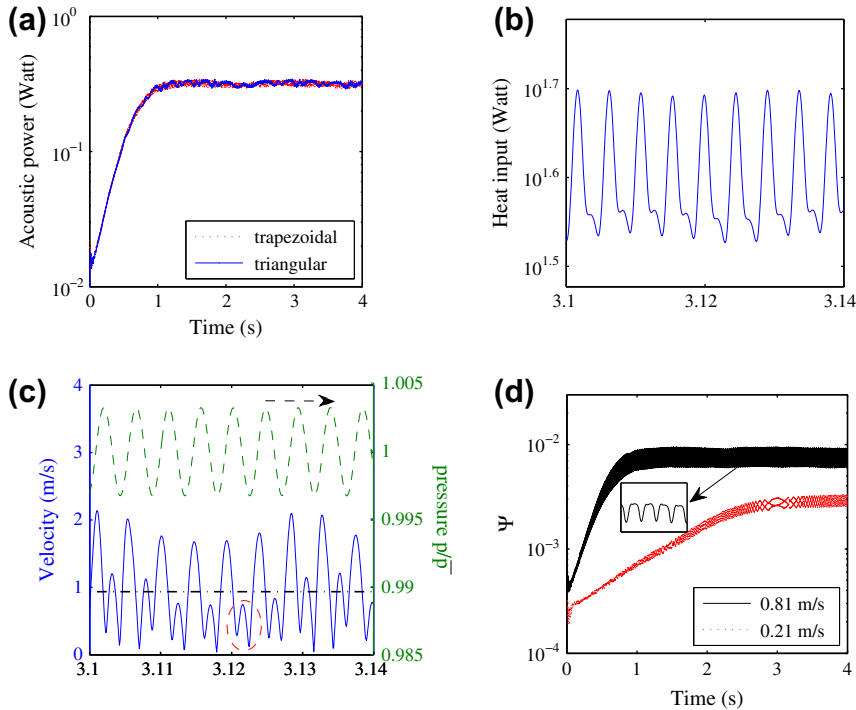


Fig. 4. Variation of acoustic power (a), heat input (b), instantaneous velocity and pressure fluctuation (c) and the overall energy efficiency (d).

$$h = \lambda \frac{\partial T_s / \partial x_i}{\Delta T_f} = \mathcal{F}(\text{Pr}, \text{Re}) \quad (24)$$

It is known that the convection coefficient is also a function of Prandtl and Reynolds number, as shown in Eq. (24). For given characteristic thermal length scales $\mathcal{L}_f$ in the fluid and $\mathcal{L}_s$ in the heating band [11], nondimensional Prandtl Pr , Reynolds number Re , and Nusselt Nu can be shown as

$$\text{Pr} = \frac{c_p \mu}{\lambda}, \quad \text{Re} = \frac{\rho \bar{u}_i D}{\mu}, \quad \text{Nu} = \frac{h \mathcal{L}_f}{\lambda} = \frac{\partial T_s}{\partial x_i} \frac{\mathcal{L}_f}{\Delta T_f} \quad (25)$$

where λ is the thermal conductivity. In our work, Pr is approximately 0.86. $\text{Re} \approx 1800 < 2300$ at the inlet of the bottom branch. Thus the inlet flow is a laminar one.

To solve these conservation equations, 2nd-order schemes are chosen for pressure interpolation and spatial discretization, while the SIMPLE algorithm is used for velocity–pressure coupling. To take the influence of temperature on the physical properties of air into account, the specific heat $c_p(T) = (\partial \mathcal{H} / \partial T)_p$, the viscosity $\mu(T) = \mu_{\text{ref}} \frac{T_{\text{ref}} + C}{T + C} (T/T_{\text{ref}})^{3/2}$ (here C is Sutherland's constant) and the thermal conductivity $\lambda(T) = c_p(T) \mu(T) / \text{Pr}$ are modeled as temperature-dependent [32]. The following values at 300 K are applied: $c_p = 1.006$ kJ/kg K, $\mu = 1.724 \times 10^{-5}$ kg/m s and $\lambda = 0.0242$ W/m K.

4. Numerical results

To gain insights on the interaction between unsteady heat release and acoustics, parametric studies are conducted. This include examining the effects of the inlet flow velocity, heat source location, and heater surface temperature on triggering thermoacoustic oscillations. These parameters are chosen, based on the facts: (1) varying the inlet velocity results in the changes of the convection coefficient $h = \mathcal{F}(\text{Pr}, \text{Re})$ and so the heat flux q , (2) the heat source location play an important role in determining the coupling between p' and q' (i.e. the phase $\angle p'q'$) and so $\mathcal{E}_a$, (3) the heater surface temperature is critical in determining the heat flux $q \propto \Delta T = T_s - T_f$. Each of the parameters is varied, while all the other parameters remain constant.

4.1. Inlet flow velocity effect

The effect of the inlet Flow Velocity at the bottom open end on the resulting thermoacoustic oscillations is first investigated. The inlet flow velocity can be varied by changing the bottom open end inlet pressure. Fig. 3 shows the variation of the pressure oscillation and heat fluxes, as the inlet velocity is set to 5 different values. It can be seen that when the inlet velocity is $v_i \leq 0.04$ m/s, the initial disturbances decay. However, as the inlet velocity is increased, limit cycles occur. And the larger inlet velocity is, the larger the resulting pressure and heat flux oscillation amplitude is.

In order to gain insight on the heat-to-sound energy conversion, the total acoustical energy can be estimated by using Eq. (12). Fig. 4(a) shows the variation of the acoustic energy with time, as the inlet velocity is set to 0.81 m/s. It can be seen that the acoustical power is increased with time by extracting more thermal energy. When a limit cycle occurs at $t \geq 1.0$ s, the acoustic power keeps almost constant. This is due to the balanced periodic energy exchange between the heat and sound.

If we define the energy conversion efficiency Ψ of converting input heat power $\mathcal{Q}_t$ into output acoustical power $\mathcal{E}_a$ as

$$\Psi = \frac{\mathcal{E}_a}{\mathcal{Q}_t} = \frac{\int_A (p' \mathbf{v} \cdot \mathbf{n}) dA}{\sum_{i=1}^3 2\pi \mathcal{R}_i H_s q'} \quad (26)$$

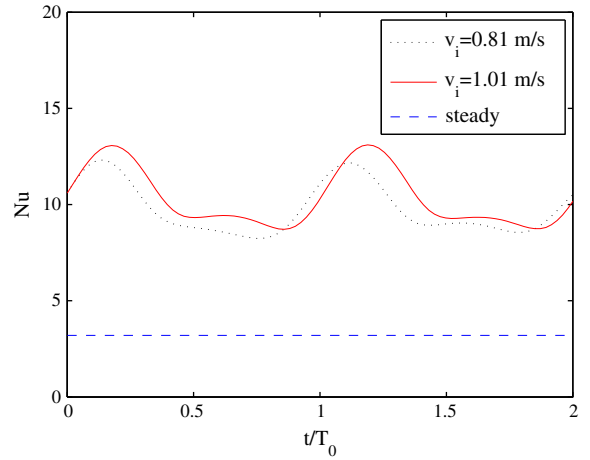


Fig. 5. Variation of Nusselt number with normalized time t/T_0 ($T_0 = 2\pi/\omega_0$).

then the fraction of heat input being converted into acoustical energy is shown in Fig. 4(d), as denoted by the solid line. Here p' denotes the pressure fluctuation, A is the cross-sectional area of the tube, H_s is the heating band height, $\mathbf{v}$ is the velocity fluctuation, $\mathcal{R}_i$ denotes the radius of the i th heating ring, and q' is the heat flux fluctuation. It can be seen that about 0.79% of total heat addition is converted into acoustical energy, which is much efficient than 0.28% when the inlet velocity is set to 0.21 m/s, as denoted by the dot line. Thus the increased pressure fluctuation amplitude with the inlet velocity, as shown in Fig. 3(a) indicates the intensified energy conversion between the heat and sound.

Flow reversal [5] would occur when the velocity perturbations exceed the mean. This is accompanied by large fluctuations in heat

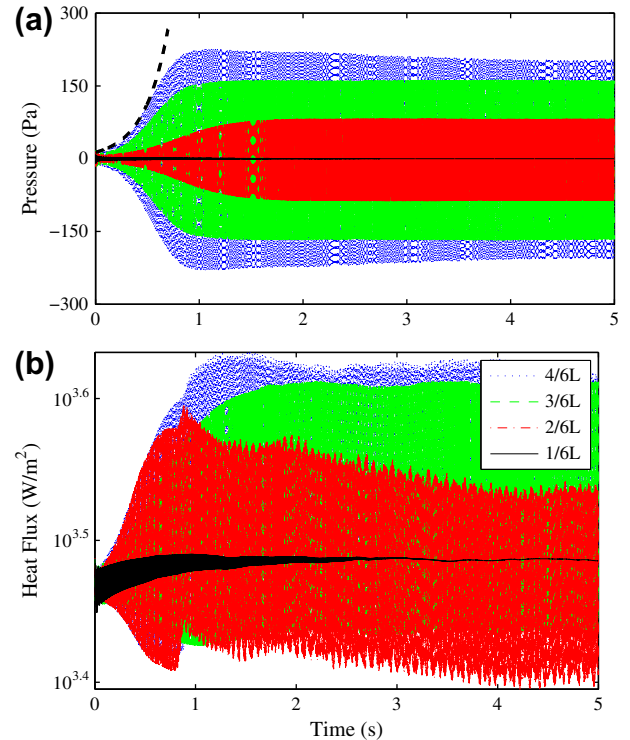


Fig. 6. Variation of pressure and heat flux fluctuations, as $T_s = 1100$ K, $p_i = 0.3$ Pa, $p_o = 0$ Pa, the heater is placed at 4 different locations and the corresponding mean flow velocities at the bottom branch inlet are 0.30 m/s, 0.56 m/s, 0.84 m/s and 0.96 m/s: (a) pressure fluctuations (b) heat flux fluctuations.

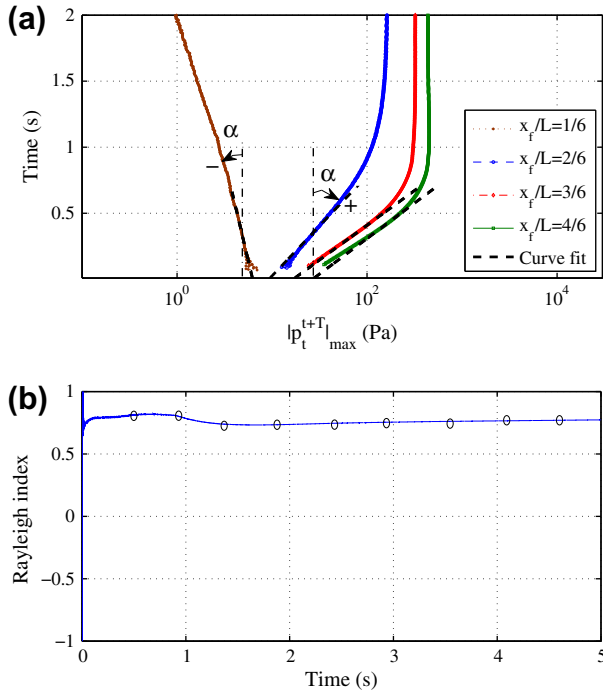


Fig. 7. (a) Variation of the mode growth rate σ , as the heater is placed at 4 locations, (b) Rayleigh index between $q(t)/\bar{q}$ and $p(x_f, t)/\bar{p}$, as $x_f/L = 3/6$.

release rate. When the reverse flow occurs, the working fluid is heated part of the way to the heater temperature on its first pass through the heater, and receives more energy on its second pass. The variation of the instantaneous velocity between two neighboring heating bands is shown in Fig. 4(c). The dash line denotes the mean flow speed. It can be seen that the fluctuating part is larger than the mean part. Furthermore, the presence of the spurs in the velocity indicates the flow reversal and nonlinearity of the thermoacoustic system. Similar nonlinearity behavior is observed in the thermal power added to the system, as shown in Fig. 4(b). However, this is different from the pressure fluctuations, as denoted by the dash line.

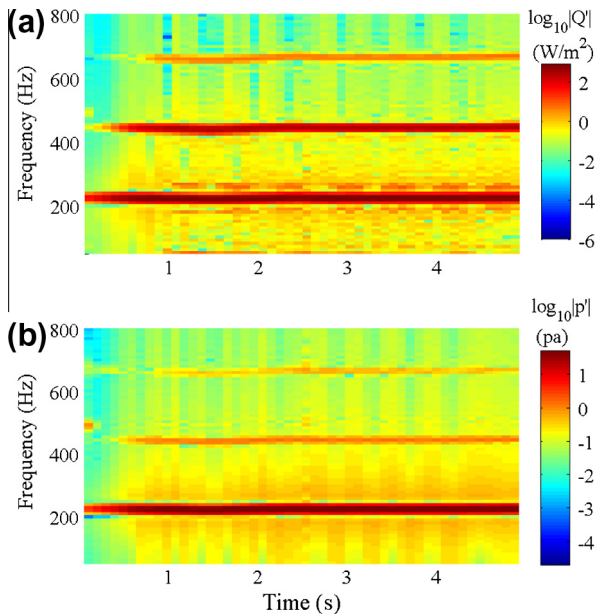


Fig. 8. Spectra of the heat flux (a) and the pressure fluctuation (b), as $x_f/L = 3/6$, $T_s = 1100$ K, $p_i = 0.2$ Pa.

As the inlet flow velocity is changed, Nusselt number is found to vary as shown in Fig. 5. It can be seen that with inlet flow velocity increased, the mean Nu number is increased. This reveals the dominant role of convection over the conduction. Furthermore, compared with the steady case (corresponding to no thermoacoustic oscillations present) as denoted by the dash line, the presence of thermoacoustic oscillations results in significant increase of heat transfer, which has the potential for energy efficient drying.

4.2. Heater location effect

The effect of the heater location on the resulting thermoacoustic oscillations is then investigated. Fig. 6 shows the variation of the pressure and heat flux fluctuations, as the heater is placed at 4 different locations. It can be seen that when $x_f/L = 1/6$, initial flow disturbances decay. However, as x_f/L is increased, the amplitudes of the pressure oscillation and heat flux are increased, and limit cycles eventually occur. This is due to the intensified energy conversion from heat to sound. In addition, the growing of the pressure oscillations at $0 \leq t < 1.0$ as shown in Fig. 6(a) can be approximated by using the real part of a complex exponential function

$$\mathcal{G}(\omega, \sigma) = \Re\{\mathcal{A}_0 e^{i\omega t + \sigma t}\} \quad (27)$$

where $\mathcal{A}_0$ denotes the initial amplitude at $t = 0$, ω denotes the oscillation mode frequency and σ describes the mode growth rate. The thick dash line in Fig. 6(a) describes $\Re\{26.67 e^{i2\pi 216t + 4.287t}\}$. Comparing it with the amplification of the pressure oscillations as $x_f/L = 4/6$ reveals that the exponential function can approximate the growing process very well when $t < 0.55$ s. However, if the growth rate σ keeps constant, the approximation at $t > 0.55$ s does not agree with the pressure fluctuation growing. This indicates that the growth rate σ is varied during the amplification process. It is worth noting that the heat flux amplification cannot be simply approximated by using an exponential function due to its inherent nonlinearity as discuss later.

The time-varying growth rate of the dominant mode of the pressure oscillations is shown in Fig. 7. It can be seen that the

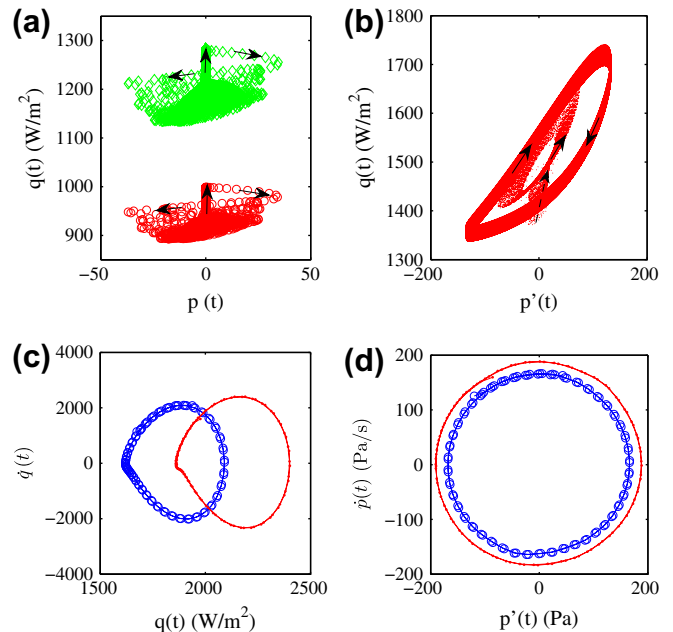


Fig. 9. Phase-space plots for the pressure oscillations and heat fluxes (a) phase diagram between $q(t)$ and $p'(t)$, $\diamond$ denotes $T_s = 700$ K, and $\circ$ denotes $T_s = 600$ K, (b) phase diagram between $q(t)$ and $p'(t)$ $T_s = 800$ K (c) phase diagram between $\dot{q}(t)$ and $q(t)$ and (d) phase diagram between $\dot{p}(t)$ and $p'(t)$: $\circ$ denotes $T_s = 900$ K, and $\times$ denotes $T_s = 1000$ K.

Table 1Measured growth rate and frequency of the dominant thermoacoustic mode, as $x_f/L = 3/6$ and $p_i = 0.1$ Pa.

Heater temperature (K)	600	700	800	900	1000
Mode growth rate	$-17.67 \rightarrow 0$	$-9.00 \rightarrow 0$	$0.73 \rightarrow 0$	$1.33 \rightarrow 0$	$1.86 \rightarrow 0$
Mode Frequency (Hz)	NA	NA	214.4	216.38	218.82

growth rate is $\sigma = \tan \alpha < 0$ and the pressure fluctuation decays at $x_f/L = 1/6$, since α is negative (anti-clockwise). However, when the heater is moved to $x_f/L \geq 2/6$, the mode growth rate $\sigma = \tan \alpha > 0$ and the pressure oscillations are amplified. Furthermore, $\sigma(x_f/L = 2/6) \leq \sigma(x_f/L = 3/6) \leq \sigma(x_f/L = 4/6)$. The larger σ mean that initial triggering disturbances will grow into a limit cycle more quickly. Furthermore, $\sigma \rightarrow 0$ with $t \rightarrow \infty$, and the mode growth rate is saturated.

Rayleigh index $\mathcal{R}_A$ is generally used as a critical indicator of the coupling between heat and sound [5,7,30]. It describes the energy exchange intensity between unsteady heat release and the pressure oscillations. If $\mathcal{R}_A > 0$, the acoustical energy in the present system is increased and tends to drive the unstable process into a limit cycle. However, a negative $\mathcal{R}_A$ indicates a ‘destruction’ interaction between unsteady heat release and pressure oscillations and so the thermoacoustic oscillations decay with time. Thus it would be interesting to check the Rayleigh index $\mathcal{R}_A$. Following the work [24], we defined a normalized Rayleigh index $\mathcal{R}_A$ as given as

$$\mathcal{R}_A(t) = \frac{\iint \delta(\mathbf{x} - \mathbf{x}_f) d\mathbf{x} \int_0^t p'(\omega, \psi) q'(\omega, \psi) d\psi}{\sqrt{\int_0^{2\pi/\omega} p'^2(\omega, \psi) d\psi} \sqrt{\int_0^{2\pi/\omega} q'^2(\omega, \psi) d\psi}} d\psi \quad (28)$$

Fig. 7(b) shows the variation of the Rayleigh index $\mathcal{R}_A(t)$ with time, as $x_f/L = 3/6$. It can be seen that when initial perturbation grows into a limit cycle, Rayleigh index $\mathcal{R}_A$ is positive, keeps increasing and finally ‘saturated’ at approximately 0.90. This verifies our numerical model and explains why self-sustained thermoacoustic oscillations occur in the T-shaped system.

Fig. 8 shows the time–frequency analysis of the heat flux $q'(t)$ and the pressure oscillations $p'(t)$. It can be seen that the spectra of the heat flux consists of one dominant frequency at approximately 220 Hz and two harmonics with large amplitude. However, the harmonics are negligible in the pressure spectra. This confirms that acoustic waves can be approximated by using linearization analysis. However, nonlinearity is mainly arising from the unsteady heat release.

4.3. Heat flux effect

As shown in Eq. (13), the heat flux $q(t) \propto (T_s - T_f)$ plays an important role in generating thermoacoustic oscillations. To investigate its effect, the heater temperature T_s is set to 5 different values and so the mean heat addition $\bar{q}$. Fig. 9 illustrates the variations of the heat flux $q(t)$ and pressure fluctuations $p'(t)$, as the heating bands are set to different temperatures. It can be seen that the mean heat flux is increased with increased heater temperature. Furthermore, the instantaneous heat flux $q(t)$ and pressure fluctuations $p'(t)$ decay, as the heater temperature is lower than $T_s = 700$ K. However, as the temperature is above 800 K, as show in Fig. 9(b)–(d), the nonlinear thermoacoustic oscillations as indicated by the circle-shaped diagrams are generated. And the amplitudes of the heat flux and the pressure fluctuations are increased with the heater temperature. Moreover, the main nonlinearity is identified in the rate of heat release, as revealed by comparing the phase diagrams of the heat fluxes (pomelo-like) and pressure fluctuation (circle-shaped).

In addition, the growth rate of the dominant thermoacoustic mode is calculated and shown in Table 1. The negative mode

growth rate means the pressure and heat flux fluctuations are decaying. And the system is stable with no thermoacoustic oscillations generated. However, when the growth rate is positive, the fluctuations are amplified and finally grow into a limit cycle. The frequency of the dominant mode is found to slightly shifted, as

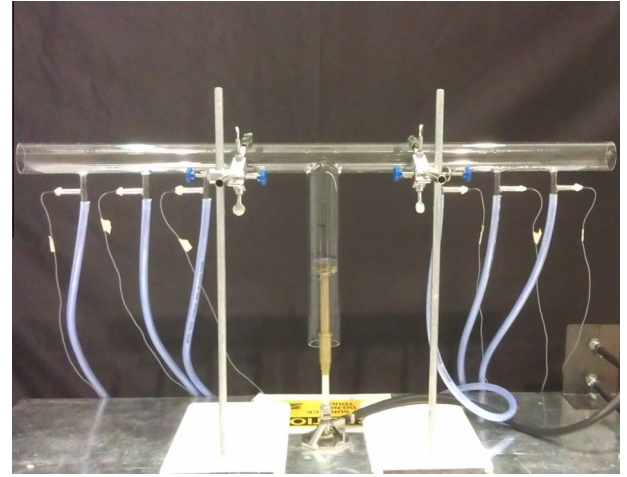


Fig. 10. Experimental setup of T-shaped thermoacoustic system with a fine metal gauze attached on top of a Bunsen burner.

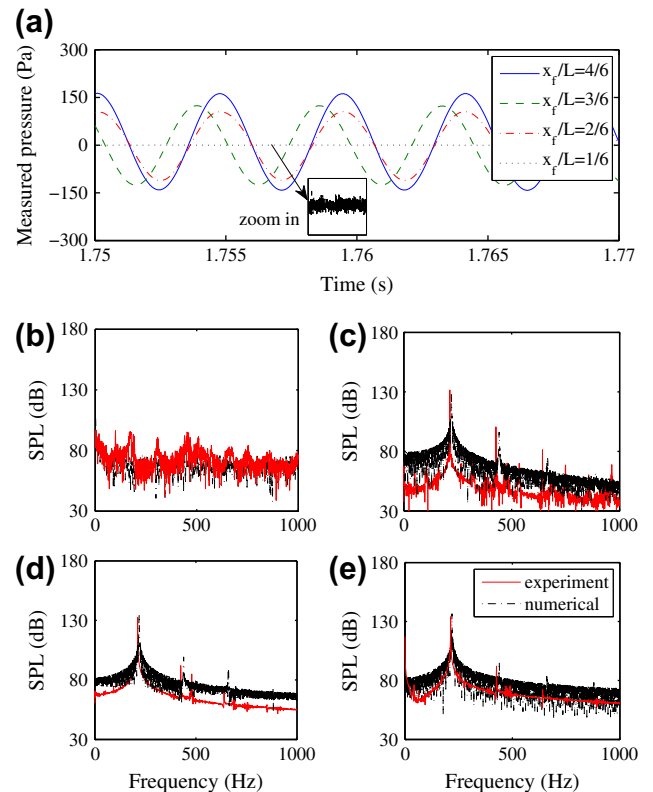


Fig. 11. Measured pressure oscillations, as the fuel flow rate is set to 230 ml/s, and the heat source is placed at 4 different locations (a) time-domain measurement, (b)–(e) frequency spectra at $x_f/L = 1/6$, $x_f/L = 2/6$, $x_f/L = 3/6$ and $x_f/L = 4/6$ respectively.

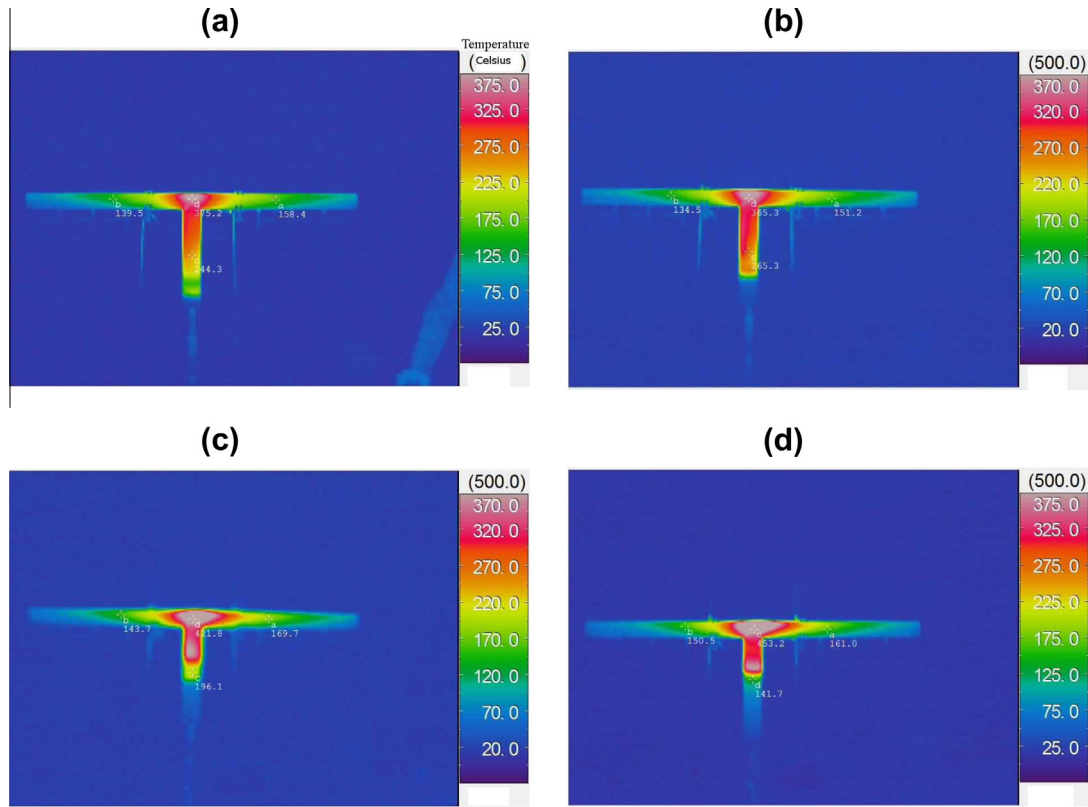


Fig. 12. Temperature contour measurements by using an infrared camera, as the heat source is placed at 4 different locations.

the heater temperature is set to different values. This might be due to the variation of the sound speed $c \propto \sqrt{T}$.

5. Description of experiment

5.1. Experimental setup

In order to validate our numerical findings, a T-shaped tube with a premixed laminar flame confined is designed and tested. It is made of quartz glass. And it has a mother tube (bottom stem), which splits into two daughter tubes (i.e. horizontal branches) with equal lengths, see Fig. 10. T-shaped junctions protruding from both bifurcating branches are designed for placing microphones. To measure pressure fluctuations along the setup, two arrays of B&K microphones (Type 4794) are implemented: each array is

involved with 3 microphones. The geometry, dimensions of the setup and the flow parameters are shown in Appendix.

5.2. Experimental results

5.2.1. Parametric measurements

Experimental measurements are conducted in two sets, corresponding to our numerical simulations. One set of measurements are performed by placing the flame at 4 different locations, as

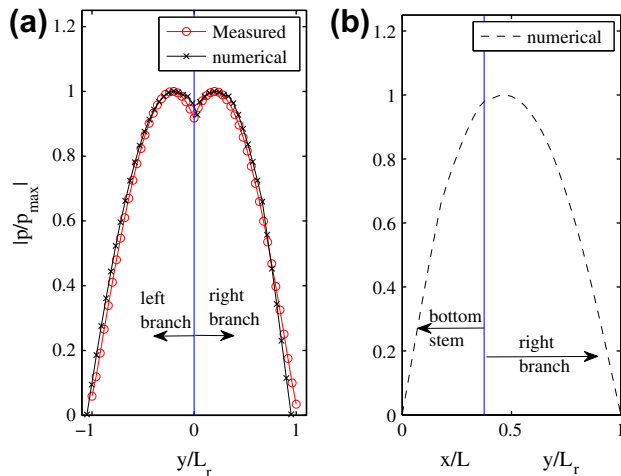


Fig. 13. Comparison of the measure and predicted mode shapes.

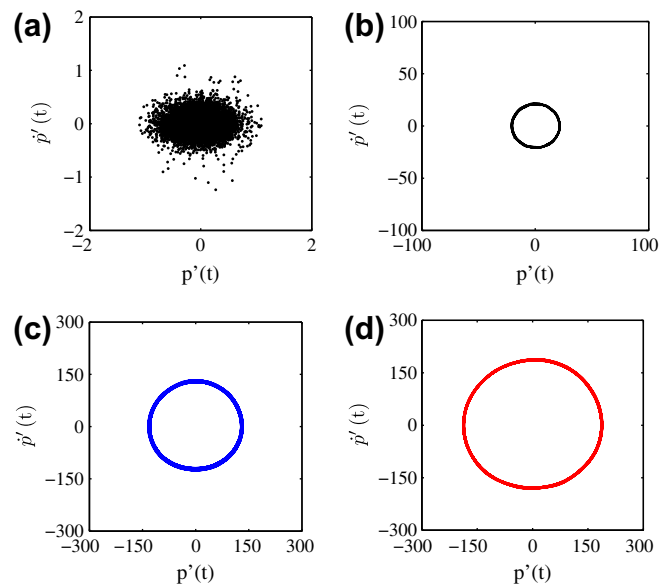


Fig. 14. Measured pressure oscillations, as the fuel flow rate is set to 4 different values: (a) 150 cc/min, (b) 200 cc/min, (c) 230 cc/min and (d) 260 cc/min.

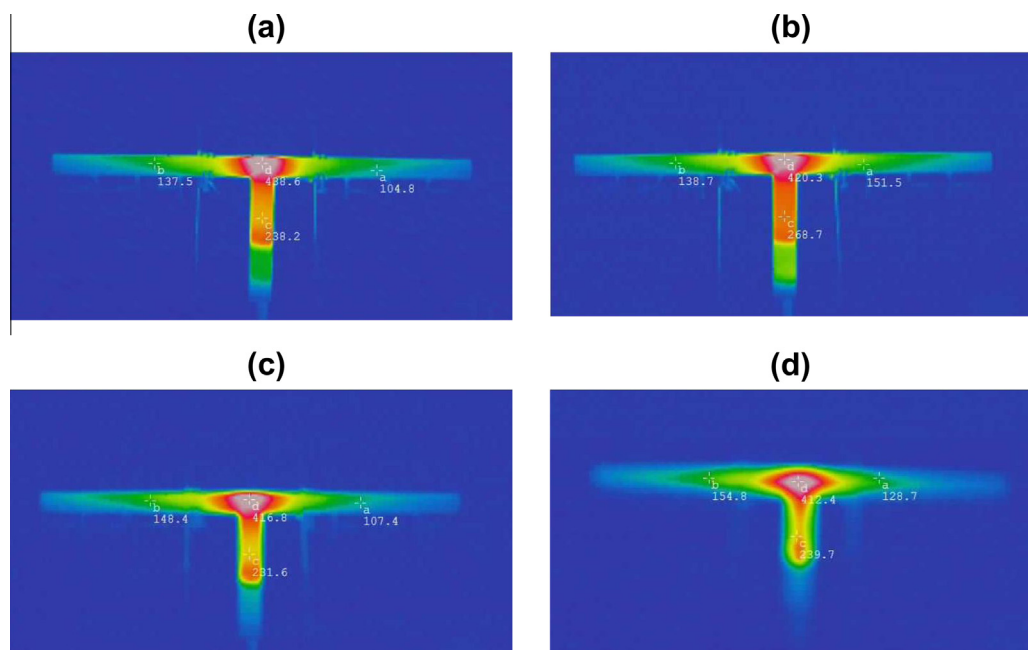


Fig. 15. Temperature contour measurement by using an infrared thermal imaging camera, as the fuel flow rate is set to 4 different values.

shown in Fig. 11(a) in time domain. Fig. 11(b)–(e) shows the frequency spectra of measured and predicted pressure fluctuations. It can be seen from Fig. 11(a) that thermoacoustic oscillations could be generated, depending on the flame location. Correspondingly, the mean flow velocity resulting from natural convection at the inlet of the bottom stem is increased from 0.04 m/s, 0.10 m/s, 0.17 m/s to 0.25 m/s as measured by using a thermal anemometer TA5, as the flame location is varied from 1/6 to 4/6. When a limit cycle is produced, the maximum pressure oscillation can reach about 136 dB, as shown in Fig. 11(b). The dominant mode is at around 216 Hz and it has multiple harmonics (indicating nonlinearity) as shown in the graphs (c)–(e). Comparison of the experimental measurements and the numerical predictions shows good qualitative agreement in terms of the sound pressure level (SPL) and the mode frequencies.

The temperature along the T-shaped system is measured by using an infrared camera [4,6]. It captures the quartz tube surface temperature spectrum. However, it can represent the 'hot' status of the sound waves inside each branch. The camera is set to operating range of 0–500° Celsius (corresponding to 273–773 K). Fig. 12 shows the measured temperature contour. It can be seen that there is a local maximum temperature at the junction section. And the temperature is decreased along the axis of the horizontal branch. The measured temperature contour (mushroom-shaped) is similar as our numerical prediction as shown in the top graph of Fig. 2.

To gain insight on the thermoacoustic oscillations, the mode shapes along the horizontal branches are measured, as shown in Fig. 13(a). It can be seen that pressure nodes are located at each ends. And the maximum pressure fluctuation occurs at approximately 0.4 m away from the open ends. Comparing the measured mode shape with the numerical predicted one, good agreement is obtained. Fig. 13(b) shows the mode shape along the bottom stem and the right branch. The total length is corresponding to half of wave length of the thermoacoustic oscillations at approximately 220 Hz. It is worth noting that the mode shapes are measured by using the classical two-microphone technique [33]. The sound speed is obtained by using the mean value of the local temperature measurements. And the mean flow speed is neglected.

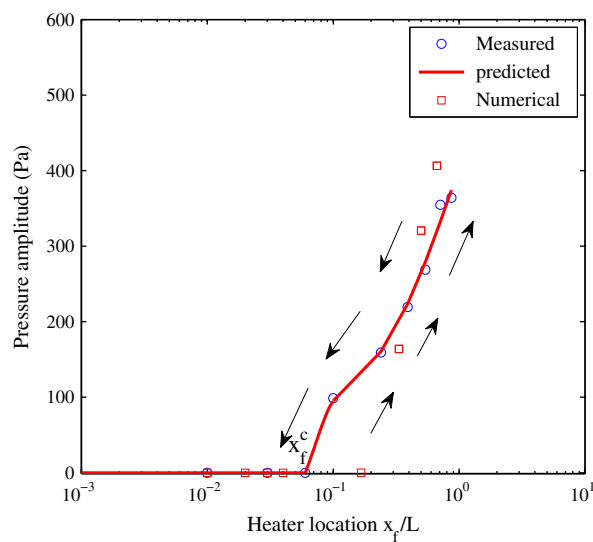


Fig. 16. Hopf bifurcation diagram of measured and numerical predicted thermoacoustic oscillations, as the fuel flow rate is set to 300 cc/min.

The other set of experiments is to investigate the effect of heat flux. This is achieved by varying the fuel flow rate. Fig. 14 shows the measured pressure oscillations, as the fuel flow rate is set to 4 different values. It can be seen that when the fuel flow rate is lower than 150 cc/min, no limit cycle is generated. This is most likely due to the fact that the thermal energy input is not larger than that dissipated due to viscous and thermal effects. The temperature contour measurement as shown in Fig. 15 is similar to those as shown in Fig. 12. However, the local maximum temperature at the junction section is decreased from 711.75 K to 685.55 K, as the fuel flow rate is increased. This is most likely due to the intensified heat-to-sound conversion (see Fig. 16).

5.2.2. Supercritical Hopf bifurcation

A bifurcation diagram in terms of the pressure fluctuation amplitude can be obtained by varying the heater location gradually while all the other parameters remain constant. The nonlinear behavior around the Hopf bifurcating point x_f is characterized by a gradually increase of the amplitude as $x_f > x_f^*$. For $x_f < x_f^*$, all perturbations imposed on the thermoacoustic system tend to decay to zero.

In general, the measured results are in good agreement with the numerical predictions, in terms of the mode shapes, mode frequencies and the supercritical Hopf bifurcation diagram. This reveal that the numerical model developed can be used to study the onset of thermoacoustic oscillations and facilitate the design of convection-driven standing-wave heat engine systems, which might be utilized for energy harvesting or drying.

6. Discussion and conclusions

In summary, a nonlinear T-shaped standing-wave thermoacoustic system is numerically and experimentally investigated to gain insights on the energy conversion process from heat to sound. The physical processes of the generation and propagation of sound waves by inputting thermal energy is mathematically illustrated by performing a generalized thermodynamic analysis. Three key parameters are identified and examined: (1) inlet flow velocity, (2) the heater location, and (3) heater surface temperature. Their effects on triggering limit cycle oscillations are first investigated in 2D numerical model by varying one parameter, while all the other parameters remain constant. The main nonlinearity is identified in the heat fluxes.

To characterize the transient (growing) behavior of the pressure fluctuation, the thermoacoustic mode growth rate is defined and calculated. It is found that the growth rate decreases first and then 'saturates'. Similar behavior is observed in examining the slope of Rayleigh index, which is widely used as an onset/triggering indicator to characterize the phasing between unsteady heat release and sound. The transient behavior of heat flux cannot be simply described by a growth rate due to its nonlinearity. In addition, the overall efficiency of converting the input heat power into acoustic power is estimated and compared. It is found that the energy conversion efficiency can be increased by increasing the inlet flow velocity.

To validate our numerical findings, a cylindrical T-shaped duct made of quartz-glass with a metal gauze attaching on top of a Bunsen burner is designed and tested. To monitor the temperature and acoustic fields in the bifurcating branches, an infrared thermal imaging camera and two arrays of microphones are used. As a premixed laminar flame is placed in the bottom branch of the tube, self-sustained thermoacoustic oscillations might be generated, depending on the 3 identified critical parameters. It is shown that Hopf bifurcation is associated with the thermoacoustic system.

Table 2
Geometry and flow parameters of the T-shaped thermoacoustic system.

Geometry	Value
Left branch length, L_l (m)	0.5
Right branch length, L_r (m)	0.5
Bottom branch length, L (m)	0.3
Cross-sectional area of left branch, A_l (m ²)	0.0016
Cross-sectional area of right branch, A_r (m ²)	0.0016
Cross-sectional area of bottom branch, A (m ²)	0.002
Inlet temperature, T_u (K)	298
Inlet and outlet mean pressure, $\bar{p}_u = \bar{p}_d$ (Pa)	1.01325×10^5
Fuel,	Propane
Lower Heating value, $\bar{Q}$ (MJ/kg)	46.35

Comparison is then made between the experimental measurement and numerical predictions. Good quantitative agreement is obtained in terms of the mode shape, mode frequency, sound pressure level (SPL) and the behavior of supercritical bifurcation.

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Appendix A

The geometry, dimensions of the setup and the flow parameters are shown in Table 2.

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